Diffusion Simulation by the most simple Finite Difference Method

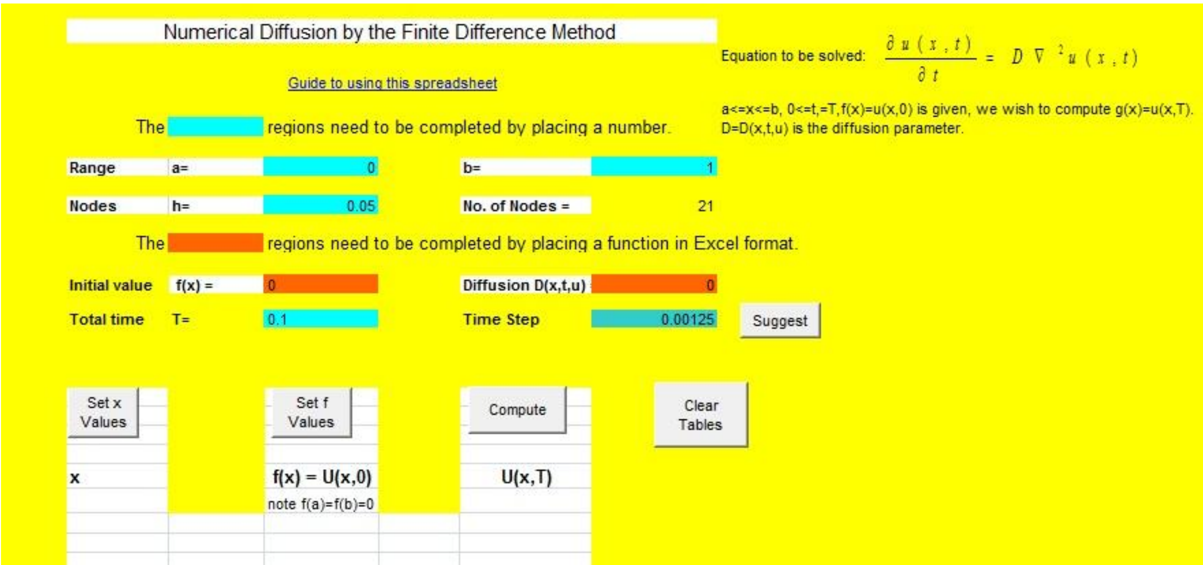
A practical demonstration in Excel[1]

This document contains a brief guide to using an Excel spreadsheet for solving the diffusion equation by the finite difference method. The equation that we will be focusing on is the one-dimensional simple diffusion equation

$$\frac{\partial u(x,t)}{\partial t} = D\frac{\partial^2 u(x,t)}{\partial x^2},$$

with $0 \le t \le T$ and $u(x,0) = f(x)$ is the initial condition and the goal is to find the solution for $0 < t \le T$ and in particular the final solution $u(x,T)$. We also require information about the solution at the edges of the region; $x=a$ and $x=b.$ The coefficient $D(\geq 0)$ represents the rate of diffusion. In this document a finite difference method for solving the diffusion equation[2] is implemented on an Excel spreadsheet[3].

The spreadsheet is shown in the following image. The spreadsheet contains macros, so the first step is to enable macros by pressing the Options... button at the top of the screen. To start a new simulation, the Clear Tables button can be used to clear the columns of input and output.



The light blue boxes need to be completed.

---

[1] Microsoft Excel

[2] Solution of the Diffusion Equation by the Finite Difference Method

[3] Excel spreadsheet applying the FDM to the diffusion equation

The spatial range of the partial differential equation is defined by placing values for *a* and *b*. In the example below the domain chosen in [0,1].

Range	a=	0	b=	1

The spatial domain is subdivided in the finite difference method. In the next part of the spreadsheet we set the length of each subdivision *h*. Note that the full length of the domain *b-a* must be be an integer multiple of *h*. The number of spatial nodes is computed and shown. For example if for *a=0* and *b=1* and with *h=20* there must be 21 nodes.

Nodes	h=	0.05	No. of Nodes =	21

The red boxes must be completed using an Excel functions. The functions that need to be defined are the initial condition *f(x)* and the rate of diffusion *D*.

Initial value	f(x) =	0	Diffusion D(x,t,u)	0.5

For the purposes of this spreadsheet the initial function *f(x)* must take the value of zero at the extremes of the domain; *f(a)=f(b)=0.*

The example that we will introduce here has the initial condition *f(x)*=sin(πx) and in Excel this is written as SIN(PI()*X). The diffusion operator can be written as a function of space (*x*), time (*t)* or the state variable (*u*). In this example let *D=0.5.*

The simulation is run for a time domain of *[0,T].* In the next row of the spreadsheet the total time *T* and the length of the time step *k* are set.

Total time	T=	0.1	Time Step	0.0025	Suggest

In the example the total time is 0.1 and the time step is 0.0025. The larger the time step, the faster the method, but also the less accurate. Critically, the time step for the method used in this method needs to satisfy the *Courant-Friedrichs-Louis* condition. The [Suggest] button can be used to select an appropriate value for the time step. However, this can only be used when the [Set x Values] and [Set f Values] buttons have bee npressed.

The problem is now set up. To activate the solution, the first step is to set the finite difference grid and the initial conditions. This can be activated by pressing the [Set x Values] and [Set f Values] buttons. For the example this gives the following tables

Set x Values		Set f Values
x		f(x) = U(x,0)
		note f(a)=f(b)=0
0		0
0.05		0.156434465
0.1		0.309016994
0.15		0.4539905
0.2		0.587785252
0.25		0.707106781
0.3		0.809016994
0.35		0.891006524
0.4		0.951056516
0.45		0.987688341
0.5		1
0.55		0.987688341
0.6		0.951056516
0.65		0.891006524
0.7		0.809016994
0.75		0.707106781
0.8		0.587785252
0.85		0.4539905
0.9		0.309016994
0.95		0.156434465
1		0

On pressing the buttons, the columns of numbers underneath should be completed. Otherwise an appropriate message will be displayed. The *x* column will list a set of equidistant nodes between *a* and *b*.

The second column lists the initial function *f(x)*, evaluated at the nodes. If you don't want to use the function definition in Excel for the initial function *f(x)*, then the values may be punched in manually, for example, or defined as a direct function of the *x*-values column.

The simulation is activated by pressing the Compute button. The computation can take time, but once completed the results – the computed nodal values for the final solution $u(x,T)$ - are displayed in the column below. The intermediate solutions are stored on the right of the spreadsheet. The results from the example discussed are given below. Once the results have been obtained then the normal Excel graphics can be used to display the results. In the following graph the initial and final functions are shown.

Compute
U(x,T)
0
0.095308037
0.188269273
0.276594696
0.358109439
0.430806339
0.492895357
0.542847656
0.579433244
0.601751262
0.609252167
0.601751262
0.579433244
0.542847656
0.492895357
0.430806339
0.358109439
0.276594696
0.188269273
0.095308037
0